

15-day drag parameters. Tables 2 and 3 list the results after the batch filter had nearly converged using Cowell's method. Notice that the measurement rms in Table 2 exhibits relatively large, seemingly random changes from one iteration to the next indicating that the integrated solution is not properly responding to the updates from the batch filter. Due to differences in the numerical integration errors between the two methods, the initial results in Table 3 appear to degrade. This is a result of changes in the estimated values for the  $C_i$  parameters. Once the estimates for  $C_i$  converge, the measurement rms rapidly converges to the LPRMS. This is in sharp contrast to the results using Cowell's method, which required several more iterations than those listed in Table 2 before the measurement rms happened to agree with the LPRMS.

Encke's method provides a solution to the convergence problem at a relatively small cost in computer storage and time. Results obtained thus far indicate that Encke's method requires only a 4-6% increase in computer time over that required using Cowell's method. In contrast, double precision arithmetic would alleviate the convergence problem as well but would require a significant increase in computer time and inhibit the vectorization capabilities that are available on many super computers.

### Conclusions

Encke's method can be used with the batch filter to improve the convergence characteristics as well as reduce numerical integration errors. This can be accomplished for only a small increase (4-6%) in computer time. Similar results in the batch filter convergence have been exhibited in arc lengths of up to 12.8 years (approximately 31,000 orbital revolutions) and involving a large number of estimated parameters. This approach has become the standard solution technique for long arc solutions of LAGEOS, and a modified approach is being examined for use with Starlette trajectories of 3-5 years in length, which represents 15,600-26,000 orbital revolutions.

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## Experimental Modal Analysis for Dynamic Models of Spacecraft

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### Introduction

LARGE flexible appendages, such as solar arrays on spacecraft, may cause structural dynamics and control coupling problems; therefore, test-verified analytical dynamic models are required before they are placed into orbit. As space structures become larger and more flexible, ground vibration tests of the complete structure become more difficult, if not impossible. One alternative approach is the experimental component mode synthesis. The component mode synthesis technique<sup>1</sup> is well matured in the field of purely computational finite element analysis, but several obstacles must be overcome if experimentally determined substructural modal data are to be used to provide the modal information for syntheses. These problems are 1) selection of a synthesis algorithm, 2) assurance of the accuracy of the modal data of substructures, 3) measurement of the rotational displacement, and 4) evaluation of the flexibility of the connecting parts. Although scalar elements (spring, mass, and dashpot) have been successfully used in syntheses based on test results, few papers have been presented that deal with the combination of plate/beam substructures.<sup>2,3</sup> To make this a practical technique, experience must be accumulated for each type of structure.

In this Note, an unconstrained component mode synthesis technique based on measured modal data is presented. The examples are solar-array-type structures that are divided into three substructures and four flexible joints (hinges). The difficulty encountered in the synthesis of such a plate-like structure is that the rotational displacements cannot be measured in the usual modal test. This difficulty is overcome by introducing a polynomial approximation for the measured modes. The results synthesized are in good agreement with the test results obtained from the combined structures.

### Formulation

In the component mode synthesis method based on measured modal data, the unconstrained mode method is more practical because it is impossible to create an exact fixed-boundary condition for large substructures. The basic equation of the synthesis is written as

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} + [\tilde{M}_c]\{\ddot{q}\} + [\tilde{C}_c]\{\dot{q}\} + [\tilde{K}_c]\{q\} = \{0\} \quad (1)$$

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where  $[M]$ ,  $[C]$ , and  $[K]$  are diagonal matrices whose elements represent the measured modal masses, modal dampings, and modal stiffnesses, respectively, of the unconstrained components. The element of the vector  $\{q\}$  is proportional to the

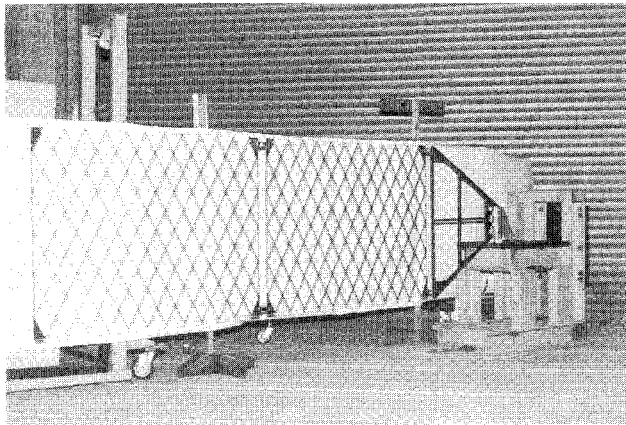


Fig. 1 Open isogrid structure (length 3.154 m, width 0.866 m, weight 3.4 kg).

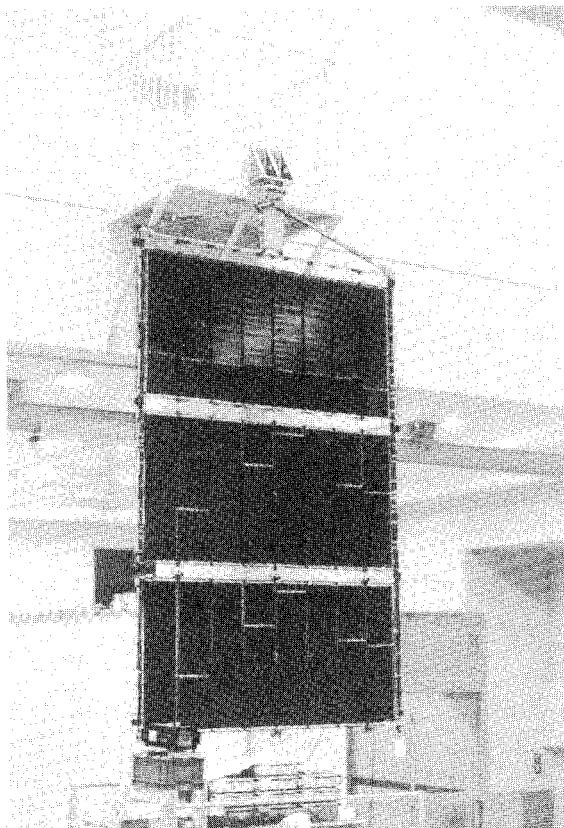


Fig. 2 Solar array of the ETS-V (length 4.177 m, width 2.322 m, weight 22.566 kg).

contribution of motion of each substructure mode. Also,  $[\tilde{M}_c]$ ,  $[\tilde{C}_c]$ , and  $[\tilde{K}_c]$  are the fully populated matrices, calculated as

$$[\tilde{M}_c] = [\Phi]^T [M_c] [\Phi], \quad [\tilde{C}_c] = [\Phi]^T [C_c] [\Phi]$$

$$[\tilde{K}_c] = [\Phi]^T [K_c] [\Phi] \quad (2)$$

where the joint matrices  $[M_c]$ ,  $[C_c]$ , and  $[K_c]$  are identified as those of an equivalent-beam finite element. Matrix  $[\Phi]$  is a mode matrix and for these three substructures is defined as

$$[\Phi] = \begin{bmatrix} \Phi_A & 0 & 0 \\ 0 & \Phi_B & 0 \\ 0 & 0 & \Phi_C \end{bmatrix} \quad (3)$$

where  $[\Phi_A]$ ,  $[\Phi_B]$ , and  $[\Phi_C]$  contain all the measured modes of the three substructures  $A$ ,  $B$ , and  $C$ , respectively.

For plate/beam structures, continuity conditions between substructures and joints have to be imposed not only on the lateral displacement but on its derivatives (rotational displacement). This is to insure that the structure remains continuous and does not kink. At each joint, therefore, the rotational displacements must be provided, but they cannot be measured in the usual modal test. The authors employed polynomial approximations over the measured lateral displacement modes in order to produce the rotational displacement. The form of

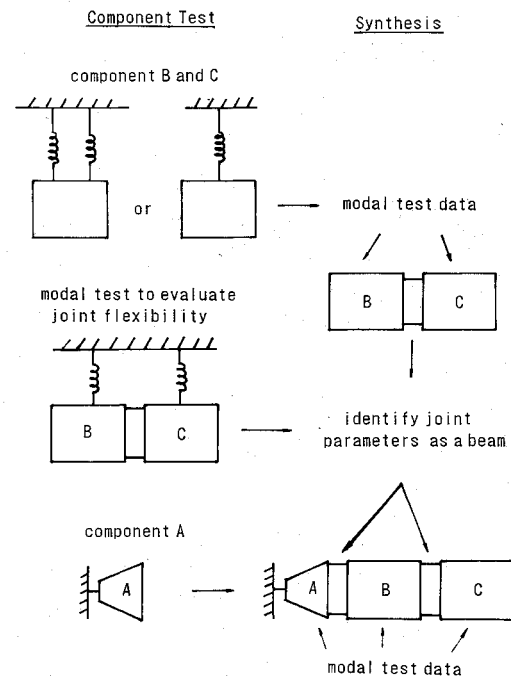


Fig. 3 Synthesis procedure.

Table 1 Component modes employed in the synthesis

Component		Open isogrid paddle	ETS- $\gamma$ paddle
A	part	stay	yoke and inner panel
		(lateral)	(lateral) (in-plane)
	modes	3 modes	7 modes 1 mode
	points measured	9	51 17
	terms of Eq. (4)	6	10 6
B,C	part	inner and outer panels	center and outer panels
		(lateral)	(lateral) (in-plane)
	modes	11 modes	7 modes 6 modes
	points measured	25	60 8
	terms of Eq. (4)	15	15 6

**Table 2 Frequency comparison for the open isogrid paddle, Hz**

Synthesis	Test	Mode shape
0.88	0.84	1st bending
3.95	3.94	1st torsion
4.21	4.16	2nd bending
11.11	11.83	2nd torsion
12.98	12.52	3rd bending
16.09	17.37	3rd torsion
19.00	20.55	4th bending

**Table 3 Frequency comparison for the paddle of the ETS-V, Hz**

Synthesis	Test		Mode shape
	Free decay	Random	
0.54	0.50	0.46	1st bending
2.03	1.22	1.20	1st torsion
1.95	2.15	2.20	1st in-plane
2.53	2.99	—	2nd bending
4.23	3.7	4.13	2nd torsion
6.08	—	—	3rd bending
8.38	—	—	2nd in-plane

the polynomials employed is

$$w = c_1 + c_2x + c_3y + c_4x^2 + c_5xy + c_6y^2 + c_7x^3 + c_8x^2y + c_9xy^2 + c_{10}y^3 + \dots \quad (4)$$

The order of the polynomial depends on the number of mode measuring points, and it does not exceed the fourth order. The constants  $c_i$  are determined by the least-square approximation method. The rotational displacements  $\partial w/\partial x$  and  $\partial w/\partial y$  then can be derived directly by differentiating Eq. (4). The matrix  $[\Phi]$  in Eq. (3) then can be enlarged by adding the terms  $\partial w/\partial x$  and  $\partial w/\partial y$  for each mode.

### Application Examples

The synthesis method based on empirical data has been applied to two actual structures. One is an open isogrid structure shown in Fig. 1. The other is the paddle of the Japanese experimental testing satellite type V (ETS-V) shown in Fig. 2. These structures are divided into three parts and four joints (hinges). Each joint is modeled as a one-beam finite element whose stiffness parameters may be experimentally determined. Even if the whole structure is large, a vibration test of the two components combined with joints is possible in most cases. The synthesis procedure used is illustrated in Fig. 3. Note that damping terms are neglected. This is because, for the structures considered, the measured damping ratio is generally not accurate enough for damping synthesis. The component modes employed in the synthesis are summarized in Table 1. For components B and C, three rigid modes are included and theoretically obtained rigid modes are used. The synthesized results are compared with experimental results for the complete structure in Table 2 and Table 3. They are in good agreement except for the first torsional mode of the ETS-V. The cause of the disagreement is not in the synthesis algorithm itself, but in the poor test results of component A because it was difficult to secure a footing for excitation at a high ground in the test of component A of the ETS-V.

### Conclusions

The component mode synthesis method based on measured modal data has been presented. The difficulty to provide the rotational displacements encountered in the synthesis is overcome by introducing polynomial approximation over the measured lateral displacement modes. The examples that have been presented demonstrate that the present method has potential as an alternative for modal tests of a complete structure.

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## Dynamic Evaluation of the NASA-ORNL Traction-Drive Joint

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### Introduction

SINCE conventional motors provide speeds that are too high for most robotic tasks and since high driving torques are desirable, speed reducers such as gear transmissions, timing belts, and sprocket and chain devices are usually incorporated at the manipulator joints. Gear transmissions can introduce backlash, which can result in low stiffness, degraded accuracy and repeatability, accelerated wear, noise and vibration, and dynamic and control problems, including limit cycle response. Backlash can be reduced using special gear designs. Harmonic drives, for example,<sup>1</sup> incorporate preloading at the tooth mesh region, but this can increase friction and local stresses. Direct-drive manipulators use high-torque, low-speed dc motors without gear reducers.<sup>2,3</sup> They are known to have low levels of joint friction and practically no backlash. Unfortunately, however, a direct-drive joint tends to be considerably heavier than a conventional joint having comparable capabilities. This would demand stronger and heavier links with associated reductions in bandwidth and increased flexibility problems.

Recently, the use of traction (friction) drive has been proposed as an alternative to gear transmission, and a manipulator using traction drives for the joints is being developed.<sup>4</sup> This drive promises improvements in the manipulator performance in terms of accuracy and efficiency. In particular, backlash problems would be virtually nonexistent and the frictional dissipation would be small. Furthermore, it has the potential for high stiffness and smooth operation, with overload protection naturally built into the joint through the friction-drive mechanism. However, traction drives are known to have two disadvantages. They are bigger and heavier than gear transmissions and practical experience with them is limited. This paper

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